

Functional Quantum Computing: An Optical Approach

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Abstract

A new model of quantum computing has recently been proposed which, in analogy with a classical λ -calculus, exploits quantum processes which operate on other quantum processes. One such quantum meta-operator takes N unitary transformations as input, coherently permutes their ordering, and outputs a new composite operator which can be applied to a quantum state. Here we propose an optical device which implements this type of coherent operator permutation. This design requires only one physical implementation of each operator to be permuted.

I. INTRODUCTION

Quantum computers [1] hold the promise of dramatic speed increases over their classical analogues for certain types of problems, such as search [2] and factoring [3]. Specific quantum algorithms are typically described using quantum circuits, which represent quantum computations at a low level, similar to representing a classical computation using machine code. Describing computations in this manner, while accurate, can be cumbersome and the resulting circuit diagrams can be difficult to understand intuitively. By developing a quantum analogue to the classical model of functional computing, it may be possible to replace the quantum circuit description with a more intuitive formalism.

Classical machine codes implement functions that take bit-value inputs and generate bit-value outputs. Although this description accurately reflects the ultimate operation of a computer, modern computer programs are rarely designed in this way. Complex computations are instead designed as a collection of functions, which return either data or other functions. Although machine code can equivalently describe a complex computation, the resulting program is often far less elegant.

In an attempt to more intuitively describe quantum computing, a new functional model has been proposed which not only simplifies the description of many quantum computations, but which also suggests a new architectural design paradigm wherein the required number of physical copies of operators—devices which realize identical operations on physical qubits—is reduced for a given quantum computation. The Q_{SWITCH} or $Q_{\text{PERMUTE}}^{(N)}$ operation [4, 5] exemplifies this functional model; $Q_{\text{PERMUTE}}^{(N)}$ takes N quantum operators as input and coherently permutes their ordering before applying the new composite operator to an input quantum bit. Here we propose an experimental, all-optical design for $Q_{\text{PERMUTE}}^{(N)}$. This design exploits a recently developed ultrafast quantum coherent switching technology [6–8] and requires only a single physical implementation of each operator to be permuted.

II. FUNCTIONAL QUANTUM COMPUTING

The original classical model of functional computing, proposed by Alonzo Church, is the λ -calculus [9]. The λ -calculus treats functions and data as the same type of objects and allows for the computation of not just functions of data, but *functions of functions*,

i.e., functions which have both functions and data as inputs and outputs. One attempt at developing a functional definition of quantum computation is the theory of “quantum combs” [10, 11], which models high-level quantum functions (i.e., quantum operators which act on other operators) as quantum circuits with sockets that quantum operators can be “plugged into”, similar to micro-chips on a circuit board. In other words, a quantum comb is a function which takes quantum operators as inputs and constructs from them a fixed quantum circuit, granting the user classical control over quantum operators. Other noteworthy proposals for functional quantum computing are [12] and [13]. A more recently proposed functional model of quantum computing extends this idea by creating circuits with coherently controlled, movable wires [5]. In this paradigm a quantum computation is modeled as a function which takes quantum operators and quantum encoded control instructions as inputs, and creates a circuit which can coherently reconfigure the connections between operators based on the control qubits, allowing a user to apply a coherent superposition of quantum operations to an input qubit state. This new model offers full quantum control over both data and operators and, due to its functional structure, allows for a more elegant description of certain problems.

In this new functional model, a proof-of-principle quantum operation has been given which has an elegant functional definition but a complex circuit representation [5]. This example operation demonstrates the functional representation’s ability to intuitively describe quantum computations. In addition, a visual representation of this functional operator suggests a practical method to conserve expensive physical resources, thereby significantly improving on a direct circuit-based implementation. This functional quantum operator takes as input N quantum operators, U_0, U_1, \dots, U_{N-1} , a register of control qubits, and a register of input qubits. It coherently orders the input quantum operators based on the value of the control qubits, creating a meta-operator—a superposition of quantum operators in (potentially) many different orderings—that is then applied to the input qubit register [5]. This operation was originally called Q_{SWITCH} [4], but is referred to as $Q_{\text{PERMUTE}}^{(N)}$ in this paper in order to avoid confusion with the all-optical switches introduced in the next section. To illustrate the differences between the circuit-based and functional descriptions, consider the simplest case: $Q_{\text{PERMUTE}}^{(2)}$.

The $Q_{\text{PERMUTE}}^{(2)}$ operator takes the following inputs: two quantum operators U_0 and U_1 ,

a control qubit $|c\rangle = \alpha|0\rangle + \beta|1\rangle$, and an input qubit $|\psi\rangle$. The output of $Q_{\text{PERMUTE}}^{(2)}$ is

$$Q_{\text{PERMUTE}}^{(2)}(U_0, U_1, |c\rangle, |\psi\rangle) = \alpha|0\rangle \otimes U_0 U_1 |\psi\rangle + \beta|1\rangle \otimes U_1 U_0 |\psi\rangle, \quad (1)$$

a coherent superposition of the operators being applied to $|\psi\rangle$ in both possible orderings. The $Q_{\text{PERMUTE}}^{(2)}$ operation can also be described by a quantum circuit with three wires (shown in Fig. 1(a)): one for the control qubit $|c\rangle$, one for the input qubit $|\psi\rangle$, and one for an ancilla qubit $|a\rangle$. The input wire leads to the operator $U_0 U_1$, while the ancillary wire leads to the reversed operator $U_1 U_0$. The control qubit coherently exchanges the states of the input and ancilla wires via a controlled-swap operator, and the input state $|\psi\rangle$ propagates down a coherent superposition of both the input and ancilla wires, passing through both sets of operators, before being deterministically placed back in the input wire by a second controlled-swap. In this way, *the input qubit experiences the quantum operators in both possible orderings*. This visual depiction suggests two separate copies of each box interacting with one input qubit and one ancilla qubit, whereas the functional definition of $Q_{\text{PERMUTE}}^{(2)}$ describes one copy of each box, each interacting with a single qubit in one of two different orderings. For a more direct comparison to the circuit model, the functional description can be illustrated as a circuit diagram with bi-directional, movable wires. In this modified circuit (shown in Fig. 1(b)), a single copy of U_0 and U_1 occupy a single wire; a “quantum switch” controls the operator ordering. The functional case conserves resources by achieving operator permutation with a single physical copy of each box.

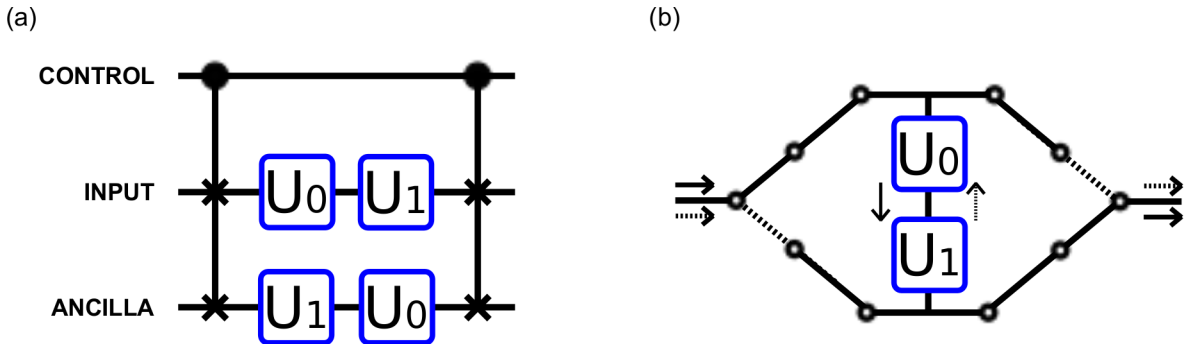


FIG. 1: (a) A quantum circuit diagram which implements $Q_{\text{PERMUTE}}^{(2)}$ using two copies of each operator. (b) A functional circuit diagram of $Q_{\text{PERMUTE}}^{(2)}$ which coherently alters the ordering of U_0 and U_1 using quantum switches to reconfigure the effective circuit. This representation uses only one copy of U_0 and U_1 .

The conservation of resources achieved by the functional definition of $Q_{\text{PERMUTE}}^{(2)}$ becomes even more significant as the problem is generalized to a permutation of N quantum operators. The $Q_{\text{PERMUTE}}^{(N)}$ operation takes as input N quantum operators, U_0, U_1, \dots, U_N , a register of control qubits $|c\rangle$, and an input register of qubits $|\psi\rangle$. It orders the unitary transformations based on the value of $|c\rangle$ —in general, a superposition of many orderings is created—and multiplies them all together into a meta-operator which is then applied to $|\psi\rangle$. In effect, this operator yields a programmable quantum computation over N quantum operators. While this functional description only references each input quantum operator once, the equivalent circuit description has been shown to require $O(N^2)$ ancillary quantum operators, $O(N^2)$ total extra qubits (including both ancillary and control), and exactly N copies of each quantum operator [5]. Mapping the functional structure of $Q_{\text{PERMUTE}}^{(N)}$ to a physical device will reduce the number of physical copies of quantum operators needed to implement it, compared to a naive implementation based directly on the quantum circuit. Below, we propose an optical device which closely simulates $Q_{\text{PERMUTE}}^{(N)}$ and realizes the operation using *only a single physical copy of each operator*.

III. AN OPTICAL OPERATOR PERMUTING DEVICE

Functional abstraction in classical computing has enabled the development of a myriad of complex programs. Now that a fully quantum-controlled model for functional quantum computing has been introduced, it is feasible that analogous development in quantum algorithms may be possible. A proof-of-principle exemplification of this functional model is the $Q_{\text{PERMUTE}}^{(N)}$ operation, which permutes the order in which N input quantum operators are applied to an input qubit. As shown above, this operation can be visualized as a quantum circuit that rewires itself using a quantum switch to coherently change the path(s) that input qubits take through the input operators. Coherent rewiring may be achieved in an optical device using quantum switches to alter the path of a polarization-encoded photonic qubit such that it encounters optical circuit elements in a particular order. Although the N input operators will act on the polarization of an input photon, the order in which the operators are applied will be coherently controlled via manipulation of the photon's spatial and temporal degrees of freedom (modes).

Recently, a switch has been demonstrated which uses an optical control pulse to couple the spatial and temporal modes of photonic qubits without otherwise disturbing their quantum state [6–8] (see Fig. 2). The two-input, two-output, all-optical, fiber-based switch is a modified nonlinear optical loop mirror (NOLM) [14] whose reflectivity is controlled via cross-phase modulation generated by an optical pump (control) pulse. In the absence of a control pulse, a NOLM can be aligned so as to perfectly reflect all incoming signal pulses. This perfect reflection is a result of destructive interference between the clockwise and counter-clockwise paths of the Sagnac loop [15]. The dual-wavelength control pulse is multiplexed in and out of the clockwise path of the Sagnac loop and consists of two equal-energy cross-polarized optical pulses such that a signal in the clockwise path of the Sagnac loop receives a polarization-independent π phase shift [16]. This phase shift turns destructive interference into constructive interference and causes any signal which passes through the pump pulse to totally transmit through the NOLM. The switch outputs are demultiplexed from the input spatial modes by fiber circulators, giving the device two input and two output spatial modes. Note that although the switching process is governed by pump light in the classical domain, the switch can act on quantum signals without disturbing them because the underlying physical process is coherent. These switches, which couple the spatial and temporal modes of photonic qubits, can be used to implement $Q_{\text{PERMUTE}}^{(N)}$.

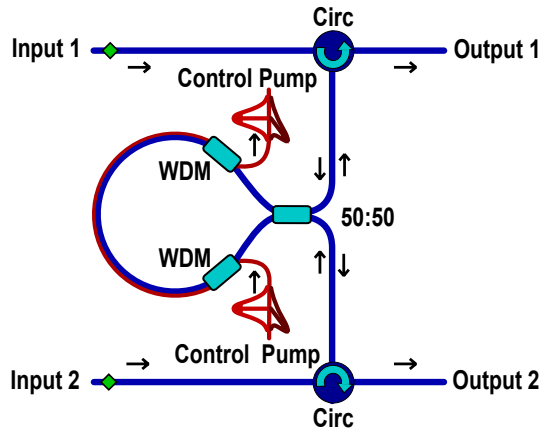


FIG. 2: A schematic for the quantum switch. Cross-phase modulation, caused by a pump pulse multiplexed in and out of a fiber loop, alters the quantum signal interference within a nonlinear optical loop mirror, thereby coherently controlling whether an input signal exits from output 1 or output 2.

The quantum switch can be thought of as having two states: “on”, where the spatial mode of an input photon is switched (i.e., an optical control pulse is present), and “off”, where the spatial mode of an input photon is not switched (i.e., no control pulse is present). In other words, the switch performs a conditional bit-flip on a photon’s spatial mode. This operation is conditioned on the presence of a control pulse. Because the photon is a quantum object, it can enter the switch as a coherent superposition of two different time modes. If the switch is “on” at one of those times and “off” at the other, the timing of the photon will coherently control the conditional spatial bit-flip; i.e., the switching action acts as a quantum controlled-NOT operation [6]. The length of the Sagnac loop determines the “on-time” of the switch—the window in which an input photon will be “switched” (the smallest demonstrated switching window is 10 ps [6]). Additionally, there is some transition time between the off and on states, approximately equal to the temporal extent of the control pulses (typically 5 ps). Using these two pieces of timing information, one can define an arbitrary number of orthogonal temporal modes (t_0, t_1 , etc...), or time-bins, in which a photon will be switched contingent on the presence of a control pulse. By constraining a single photon to exist in a coherent superposition of 2^N temporal modes

$$\sum_{i=0}^{2^N-1} \alpha_i |1\rangle_{t_i} \prod_{j=1, j \neq i}^{2^N-1} \otimes |0\rangle_{t_j} \quad (2)$$

where $\sum_{k=0}^{2^N-1} |\alpha_k|^2 = 1$, a register of N temporal qubits can be encoded onto a single photon. Additionally, a polarization qubit $|\psi\rangle$ can be encoded on the photon. Using quantum switches, it is possible to create a device which re-wires the connections between a collection of polarization quantum operators such that each temporal mode of an input photon experiences a different series of polarization operations, thereby simulating the $Q_{\text{PERMUTE}}^{(N)}$ operation.

Consider a network of $K = 2^k - 1$ switches $S_{p,q}$ connected in a binary tree structure [17], where $p \in \{0, 1, \dots, k-1\}$ indicates the level of the tree in which a switch resides and $q \in \{0, 1, \dots, 2^p - 1\}$ is the index of the switch within it’s level (see in Fig. 3(a)). Each level of the tree contains 2^p quantum switches and the tree encompasses a total of $N = K + 1 = 2^k$ spatial modes (x_0, x_1, \dots, x_{N-1}). Every switch not in the $p = k - 1$ level has its outputs connected to the inputs of two switches in the next level: $S_{p+1,2q}$ and $S_{p+1,2q+1}$. For a given p and q , a quantum switch acts on two spatial modes: $x_{2qN/2^{p+1}}$ and $x_{(2q+1)N/2^{p+1}}$, where all switches with $p > 0$ are assumed to receive a vacuum field input via the $x_{(2q+1)N/2^{p+1}}$

mode. In an $N = 8$ network, for example, $S_{1,1}$ acts on $x_{2(1)(8)/2^{1+1}} = x_{2(8)/4} = x_4$ and $x_{(2(1)+1)(8)/2^{1+1}} = x_{3(8)/4} = x_6$. Because of the tree structure, a photon input to the $S_{0,0}$ switch can be routed to any of N spatial modes at the output. In other words, the tree of quantum switches is a multiplexor which maps a single input spatial mode to one of N output spatial modes. If the structure is reversed (see Fig. 3(b)) such that the inputs to a switch $S'_{p,q}$ come from a single output of the $S'_{p+1,2q}$ and $S'_{p+1,2q+1}$ switches, then the structure becomes a demultiplexor which can route a photon input to any spatial mode of a switch in the $p = \log_2(N) - 1$ level to a single output spatial mode of the $S'_{0,0}$ quantum switch. When configured in this de-multiplexing configuration, all switches with $p > 0$ are assumed to output a vacuum field via the $x_{(2q+1)N/2^{p+1}}$ mode. To avoid confusion, S' will refer to switches used for demultiplexing and S to switches for multiplexing.

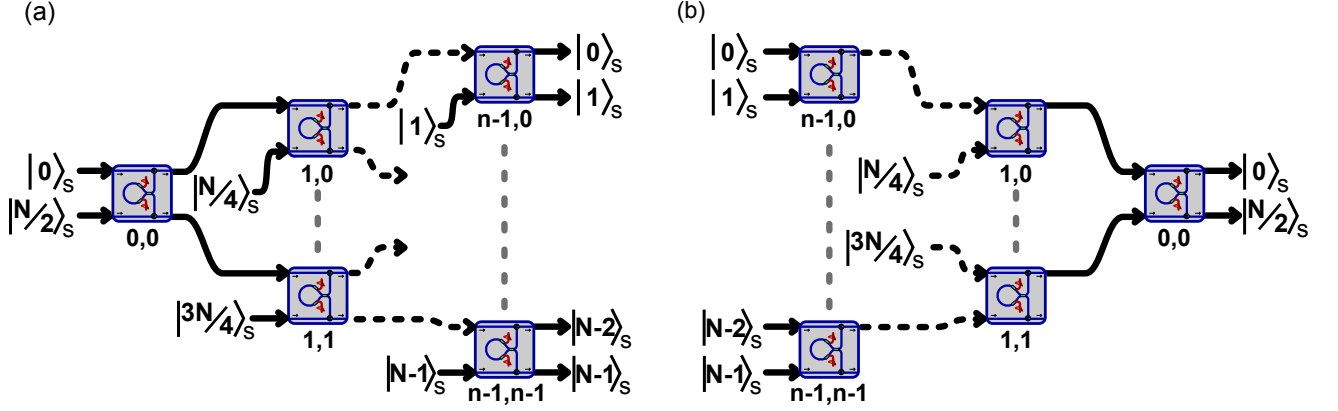


FIG. 3: (a) A network of $N - 1$ quantum switches which multiplexes an input photon from the x_0 spatial mode into a superposition of N spatial modes. (b) A network of $N - 1$ quantum switches which de-multiplexes N spatial modes of a quantum signal—in this case a single photon—into a single spatial mode (x_0).

Combining a multiplexor quantum switch network whose output spatial modes are all connected to different polarization quantum operators ($x_0 \rightarrow U_0, x_1 \rightarrow U_1, \dots, x_{N-1} \rightarrow U_{N-1}$), with a demultiplexor quantum switch network, whose inputs are all connected to the outputs of the quantum operators, yields a device which can route an input photon through any of N unitary operators and return it to its initial spatial mode (see Fig. 4). With the addition of an optical feed-back loop connecting the $x_{N/2}$ output of $S'_{0,0}$ to the $x_{N/2}$ input of $S_{0,0}$, the photon can be routed through the device an arbitrary number of times, M , passing through an arbitrary quantum polarization operator in each iteration. In other words,

the device will take an input photon via the x_0 input of $S_{0,0}$, apply one of N^M possible combinations of polarization operators, and then output the result from the x_0 output of $S'_{0,0}$. By programming the device to route the photon through a different combination of operators depending on its temporal mode, the functionality of the device can be significantly expanded.

The network of quantum switches can be programmed such that a photonic polarization qubit is routed through a unique permutation of N polarization quantum operators for each of its temporal modes. In this scheme, the state of an input photon is

$$\sum_{i=0}^{2^N-1} \alpha_i |\psi\rangle_{t_i, x_0} \prod_{j=0, j \neq i}^{2^N-1} |0\rangle_{t_j, x_0} \quad (3)$$

and the temporal modes are serialized by number and chosen such that t_{N^M-1} passes through the input switch before t_0 enters the feedback loop for the first time; the output state is

$$\sum_{i=0}^{2^N-1} \alpha_i O_i |\psi\rangle_{t_i, x_0} \prod_{j=0, j \neq i}^{2^N-1} |0\rangle_{t_j, x_0}, \quad (4)$$

where each temporal mode (t_i) has undergone a different combination of polarization operators $O_i = O_{i,M-1} O_{i,M-2} \dots O_{i,0}$, and $O_{i,k} = U_l$ where

$$l = \left\lfloor \frac{i}{N^{M-k-i}} \right\rfloor \bmod N, \quad (5)$$

and k is an integer which indexes each iteration of the photon through the device.

Each U_l can be applied to an input photon by activating a unique group of switches such that they are in the “on” state while interacting with the i th temporal mode of a qubit, and in the off state, if necessary, before the next temporal mode of the qubit will interact with them. To apply the U_l operator to a specific temporal mode of the qubit, all switches with indices such that

$$(q+1) \frac{N}{2^{p+1}} \leq l < (q+2) \frac{N}{2^{p+1}}, \quad (6)$$

and $p \neq 0$ must be turned on when that temporal mode enters the switch. Because the $p = 0$ switches also serve as input/output ports to the device, their behavior must be modified slightly. The $S_{0,0}$ switch must be turned on if either: 1) $j = 0$ and $l \geq \frac{N}{2}$, or 2) $j \neq 0$ and $l < \frac{N}{2}$. The $S'_{0,0}$ switch must be turned on if either: 1) both $j = M-1$ and $l \geq \frac{N}{2}$, or 2) $j \neq 0$ and $l < \frac{N}{2}$. By applying these switching rules for every iteration of the photon through the device, and for every temporal mode of the photon, a superposition of all N^M

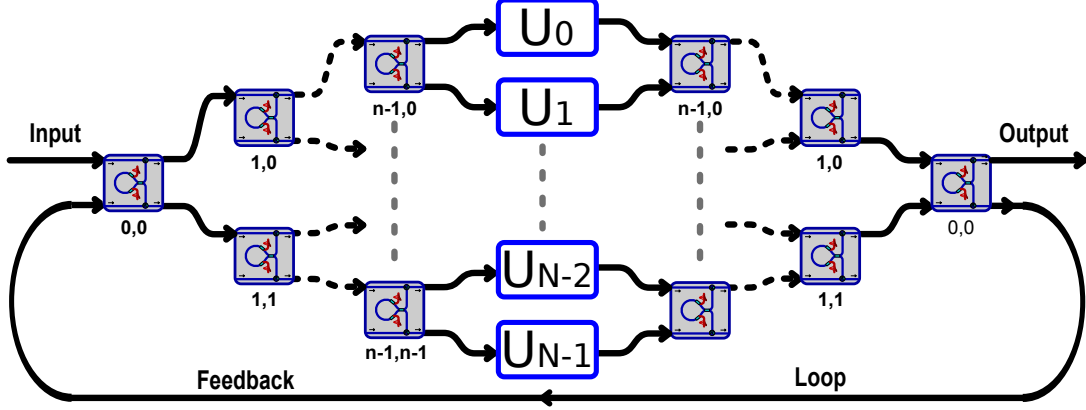


FIG. 4: An optical device which simulates $Q_{\text{PERMUTE}}^{(N)}$ using $2N - 2$ quantum switches, N polarization quantum operators, and an optical feedback loop. Because the behavior of the quantum switches is governed by a predetermined pattern of optical control pulses, the time at which a photonic qubit is input to the device can be mapped to an order in which the photon passes through the polarization quantum gates.

possible operator permutations is applied to the input polarization state $|\psi\rangle$. This device will deterministically simulate the $Q_{\text{PERMUTE}}^{(N)}$ operation if $M = N$ and the input photon is superposed over only those temporal modes to which non-degenerate operator permutations will be applied.

Consider a single quantum operator in the device, it will interact with each temporal mode of the input photon at a different time. At each of these different times, the transformation applied by the operator to the photon's polarization state may be varied depending on air turbulence, vibration of optical components, or noise in electrical components. In essence, the proposed device does not apply the quantum operator N times. Instead, it applies N different approximations of the operator. One approach to overcoming this problem is to exploit the Fourier relationship between time and frequency. Applying a narrowband frequency filter to the photon at the device output will broaden its temporal profile, though such filtering could be a significant source of loss. If sufficiently narrow filtering is possible without attenuating the signal below measurable levels, it may become impossible (even in principle) to determine whether or not the different temporal modes of the photon interacted with different approximations of each operator. In this case, every temporal mode of the photon has effectively interacted with the same approximation of each operator, and the $Q_{\text{PERMUTE}}^{(N)}$ operation has been implemented.

IV. CONCLUSION

A new model of functional quantum computation has recently been presented, one which describes quantum computers as a collection of quantum operators with programmable interconnections. Within this model, a proof-of-principle operation, $Q_{\text{PERMUTE}}^{(N)}$, has been proposed which takes as input: N quantum operators, a quantum control register, and an input quantum state; the input operators are coherently re-ordered according to the state of the control register and then applied to the input state. Here, we have proposed an all-optical device which implements $Q_{\text{PERMUTE}}^{(N)}$ on a single photon whose temporal mode provides the control qubits for the permutation and whose polarization state is used as the input quantum state. This all-optical device utilizes $2N - 2$ newly demonstrated quantum switches [6–8], and a single physical implementation of each quantum operator to be permuted. Although this device does not scale (because it requires N^M temporal modes), its implementation can drastically reduce the number of physical copies of devices used in a quantum computation, and constructing it will demonstrate that the new functional formalism is valid, opening the door to further development, and, possibly, new quantum algorithms.

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